Consensus Algorithms for Distributed Controlled FREEDM Systems

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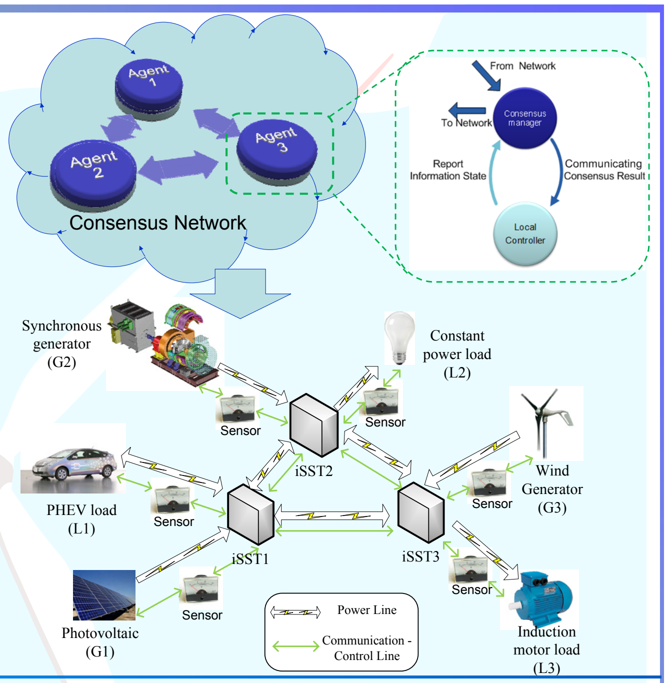
Objective:

- Provide optimized system performance with low cost(price, power, etc.) through distributed information utilizations
- Develop a consensus algorithm which Enable real-time monitoring, control and operation globally with distributed local information



Highlights

- Enabling and empowering individuals and small groups of sensors, actuators and controllers go global easily and seamlessly.
- Uniqueness– the newfound power for individuals (sensors, actuators, controllers) to collaborate/cooperate globally to solve local challenging problems

Consensus Modeling

- Local information state ξ_i
- First-order system $\dot{\xi}_i = \xi_i, i = 1,\dots,n$
- Reaching consensus $\xi_1 = \xi_2 = \dots = \xi_n$

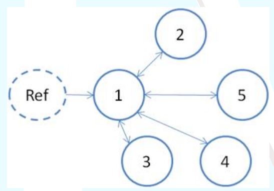
Consensus Algorithms

	Scalar Form	Matrix Form
Continuous	$u_i = -\sum_{j=1}^{n} a_{ij}(\xi_i - \xi_j), i = 1,\dots,n$	$u = -L_n \xi$
Discrete	$u_i[k] = \sum_{j=1}^{n} d_{ij}\xi_j[k], i = 1,\dots,n$	$u[k] = D_n \xi[k]$

- *A:* Adjacency matrix of a finite graph *G* on n vertices is the $n \times n$ matrix
- *L:* Graph Laplacian matrix *L=D-A, D* is the degree matrix, $D=diag(d_1,\dots, d_n)$, where d_i is the degree of a graph vertex;
- λ_2 : second-smallest eigenvalue of the Laplacian matrix of *G*. Called algebraic connectivity of *G*;
- λ_n :Largest eigenvalue of the Laplacian matrix of *G*.

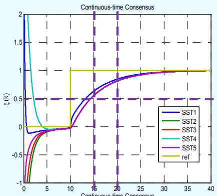
Consensus Simulation 1:

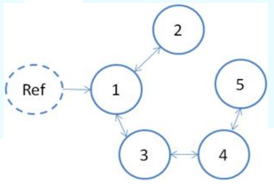
Consensus performance with different network topology (Step inputs as load references)



$$L = \begin{bmatrix} 4 & -1 & -1 & -1 & -1 \\ -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

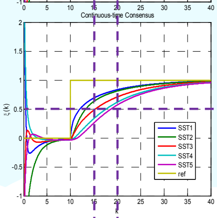
$\lambda_2 = 1$





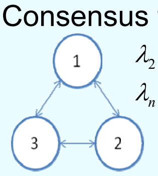
$$L = \begin{bmatrix} 2 & -1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ -1 & -1 & 2 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

$\lambda_2 \approx 0.38$



Consensus Simulation 2:

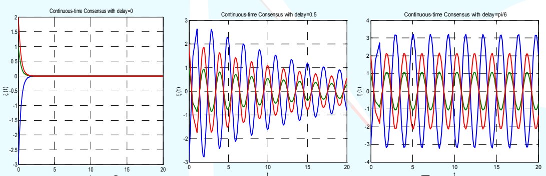
Consensus test with time-delay



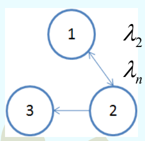
$\lambda_2 = 3$

$\lambda_n = 3$

$$L = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$



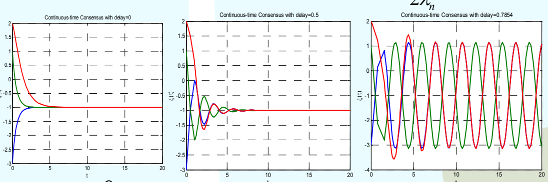
$\tau = 0$ $\quad$ $\tau = 0.5\text{sec}$ $\quad$ $\tau = \frac{\pi}{2\lambda_n} \approx 0.5236\text{sec}$



$\lambda_2 = 1$

$\lambda_n = 2$

$$L = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$



$\tau = 0$ $\quad$ $\tau = 0.5\text{sec}$ $\quad$ $\tau = \frac{\pi}{2\lambda_n} \approx 0.7854\text{sec}$

Conclusions:

When designing Time-sensitive distributed networked control system (e.g. FREEDM system) several issues need to be considered:

- Convergence rate: algebraic connectivity λ_2
- Sensitivity of time-delay: largest eigenvalue λ_n.
- The trade-off between these two parameters need to be balanced based on the requirement of application.

Future Work:

- Formulate the consensus algorithms for the FREEDM systems with both continuous time models and discrete event models
- Design high performance and reliable consensus algorithms for FREEDM systems

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